The concrete theory of numbers: initial numbers and wonderful properties of numbers repunit

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February 5, 2008

Abstract

In this work initial numbers and repunit numbers have been studied. All numbers have been considered in a decimal notation. The problem of simplicity of initial numbers has been studied. Interesting properties of numbers repunit are proved: $gcd(R_a, R_b) = R_{gcd(a,b)}$; $R_{ab}/(R_a R_b)$ is an integer only if $gcd(a,b) = 1$, where $a \geq 1$, $b \geq 1$ are integers. Dividers of numbers repunit, are researched by a degree of prime number.

Devoted to the tercentenary from the date of birth (4/15/1707) of Leonhard Euler

1 Introduction

Let $x \geq 0$, $n \geq 0$ be integers. An integer N, which record consists from n *records* of number x, we shall designate by

$$N = \{x\}_n = x \dots x, \ n > 0. \tag{1}$$

For $n = 0$ it is received $\{x\}_0 = \varnothing$ an empty record. For example, $\{10\}_3 1 = 1010101$, $\{10\}_0 1 = 1$, etc.

Palindromic numbers of a kind

$$E_{n,k} = \{1\{0\}_k\}_n 1, \tag{2}$$

where $n \geq 0$, $k \geq 0$ we will name initial numbers. We will notice that $E_{0,k} = 1$ at any $k \geq 0$.

Numbers repunit(see[2, 3, 4]) are natural numbers, which records consist of units only, i.e. by definition

$$R_n = E_{n-1,0}, \tag{3}$$

where $n \geq 1$.

In decimal notation the general formula for numbers repunit is

$$R_n = (10^n - 1)/9, \tag{4}$$

*Tarasov, B. V. The concrete theory of numbers: initial numbers and wonderful properties of numbers repunit. MSC 11A67+11B99.

where $\boldsymbol{n = 1, 2, 3, \ldots}$.

There are known only five prime repunit for $\boldsymbol{n}$ =2,19, 23, 317, 1031.

Known problem ((Prime repunit numbers[3])). *Whether exists infinite number of prime numbers repunit ?*

Will we use designations further :

$\boldsymbol{(a, b) = gcd(a, b)}$ the greatest common divider of integers $\boldsymbol{a > 0}$, $\boldsymbol{b > 0}$.

$\boldsymbol{p, q}$ odd prime numbers.

If it is not stipulated specially, the integer positive numbers are considered.

2 Initial numbers

Let's consider the trivial properties of initial numbers.

Theorem 1. *Following trivial statements are fair :*

(1) General formula of initial numbers is

$$\boldsymbol{E_{n,k} = \frac{R_{(k+1)(n+1)}}{R_{k+1}} = \frac{10^{(k+1)(n+1)} - 1}{10^{k+1} - 1}}. \tag{5}$$

(2) For $\boldsymbol{k \geq 0}$, $\boldsymbol{n \geq m \geq 1}$ *if* $\boldsymbol{n + 1 \equiv 0(mod\,(m + 1))}$, *then* $\boldsymbol{(E_{n,k}, E_{m,k}) = E_{m,k}}$.

(3) For $\boldsymbol{k \geq 0}$, $\boldsymbol{n > m \geq 1}$ *if integer* $\boldsymbol{s \geq 1}$, *exists such that* $\boldsymbol{n + 1 \equiv 0(mod\,(s + 1))}$, $\boldsymbol{m + 1 \equiv 0(mod\,(s + 1))}$, *then* $\boldsymbol{(E_{n,k}, E_{m,k}) \geq E_{s,k} > 1}$.

(4) For $\boldsymbol{k \geq 0}$, $\boldsymbol{n > m \geq 1}$ $\boldsymbol{(E_{n,k}, E_{m,k}) = 1}$ *when and only then,* $\boldsymbol{(n + 1, m + 1) = 1}$.

Proof. 1) Properties (1)—(3) are obvious.

2) The Proof of property (4). **Necessity**. Let $\boldsymbol{(E_{n,k}, E_{m,k}) = 1}$ and $\boldsymbol{(n + 1, m + 1) = s > 1}$, $\boldsymbol{s - 1 \geq 1}$. From property (3) of the theorem follows that $\boldsymbol{(E_{n,k}, E_{m,k}) \geq E_{s-1,k} = \{1\{0\}_k\}_{s-1}1 > 1}$. Appears the contradiction .

Sufficiency of property (4). Let $\boldsymbol{(n+1, m+1) = 1}$, then will be integers $\boldsymbol{a > 0}$, $\boldsymbol{b > 0}$, such that either $\boldsymbol{a(n + 1) = b(m + 1) + 1}$ or $\boldsymbol{b(m + 1) = a(n + 1) + 1}$. Let's assume, that $\boldsymbol{(E_{n,k}, E_{m,k}) = d > 1}$.

a) Let $\boldsymbol{a(n + 1) = b(m + 1) + 1}$, then $\boldsymbol{E_{b(m+1),k} = E_{a(n+1)-1,k} = (10^{a(n+1)(k+1)} - 1)/(10^{k+1} - 1) \equiv 0(mod\,E_{n,k}) \equiv 0(mod\,d)}$.

On the other hand $\boldsymbol{E_{b(m+1),k} = (10^{(k+1)\{b(m+1)+1\}} - 1)/(10^{k+1} - 1) = ((10^{b(m+1)(k+1)} - 1)/(10^{k+1} - 1)) \cdot 10^{k+1} + 1 \equiv}$
$\boldsymbol{\equiv 1(mod\,E_{m,k}) \equiv 1(mod\,d)}$. Appears the contradiction.

b) Let $\boldsymbol{b(m + 1) = a(n + 1) + 1}$, then $\boldsymbol{E_{a(n+1),k} = E_{b(m+1)-1,k} = (10^{b(m+1)(k+1)} - 1)/(10^{k+1} - 1) \equiv 0(mod\,E_{m,k}) \equiv 0(mod\,d)}$.

On the other hand $\boldsymbol{E_{a(n+1),k} = (10^{(k+1)\{a(n+1)+1\}} - 1)/(10^{k+1} - 1) = ((10^{a(n+1)(k+1)} - 1)/(10^{k+1} - 1)) \cdot 10^{k+1} + 1 \equiv}$
$\boldsymbol{\equiv 1(mod\,E_{n,k}) \equiv 1(mod\,d)}$. Have received the contradiction. □

3 Numbers repunit

Let's consider trivial properties of numbers repunit.

Theorem 2. *Following trivial statements are fair :*

(1) The number R_n is prime only if n number is prime.

(2) If $p > 3$ all prime dividers of number R_p look like $1+2px$ where $x \geq 1$ is integer.

(3) $(R_a, R_b) = 1$ if and only if $(a, b) = 1$.

Proof. Property (1) of theorem is proved in ([2, 3]), property (2) is proved in ([1]), as exercise. Property (3) is the corollary of the theorem 1. □

Theorem 3. $(R_a, R_b) = R_{(a,b)}$*, where $a \geq 1$, $b \geq 1$ are integers.*

Proof. Validity of the theorem for $(a, b) = 1$ follows from property (3) of theorem 2. Let $(a, b) = d > 1$, where $a = a_1 d$, $b = b_1 d$, $(a_1, b_1) = 1$. Let's consider equations

$$R_a = R_d \cdot \{10^{d(a_1-1)} + \ldots + 10^d + 1\},$$

$$R_b = R_d \cdot \{10^{d(b_1-1)} + \ldots + 10^d + 1\}.$$

Let

$$A = 10^{d(a_1-1)} + \ldots + 10^d + 1,$$
$$B = 10^{d(b_1-1)} + \ldots + 10^d + 1.$$

Let's assume, that $(A, B) > 1$, and q is a prime odd number such that

$$A \equiv 0 (mod\, q), \;\; B \equiv 0 (mod\, q). \tag{6}$$

If $q = 3$, then $10^t \equiv 1 (mod\, q)$ for any integer $t \geq 1$. Then from (6) it follows that $a_1 \equiv b_1 \equiv 0 (mod\, q)$. Have received the contradiction.

Thus, $q > 3$. Then there exists an index d_{min}, to which the number 10^d belongs on the module q.

$$(10^d)^{d_{min}} \equiv 1 (mod\, q),$$

where $d_{min} \geq 1$.

If $d_{min} = 1$, then it follows from (6) that $a_1 \equiv b_1 \equiv 0 (mod\, q)$. Have received the contradiction. Hence $d_{min} > 1$. As $R_a \equiv R_b \equiv 0 (mod\, q)$, then $(10^d)^{a_1} \equiv 1 (mod\, q)$ and $(10^d)^{b_1} \equiv 1 (mod\, q)$.
Then $a_1 \equiv b_1 \equiv 0 (mod\, d_{min})$. Have received the contradiction. □

Theorem 4. *Let $p > 3$ be a prime number, $k \geq t \geq 1$, $t \geq s \geq 1$ integer numbers. Then*

$$gcd(R_{p^k}/R_{p^t}, R_{p^s}) = 1. \tag{7}$$

Proof. Let's consider expression

$$A = R_{p^k}/R_{p^t} = (10^{p^t})^{p^{k-t}-1} + (10^{p^t})^{p^{k-t}-2} + \ldots + 10^{p^t} + 1.$$

If $(A, R_{p^s}) > 1$, then the prime number q exists such that

$A \equiv 0 (mod\, q)$ $R_{p^s} \equiv 0 (mod\, q)$. Hence $10^{p^t} \equiv 1 (mod\, q)$, then $A \equiv p^{k-t} \equiv 0 (mod\, q)$, $p = q = 3$. Have received the contradiction, because $p > 3$. □

Theorem 5. *Let $a \geq 1$, $b \geq 1$ are integers, then the following statements are true :*

(1) If $(a, b) = 1$, then

$$gcd(R_{ab}, R_aR_b) = R_aR_b. \tag{8}$$

(2) If $(a, b) > 1$, then

$$R_aR_b/R_{(a,b)} \leq gcd(R_{ab}, R_aR_b) < R_aR_b. \tag{9}$$

Proof. 1) Let $(a, b) = 1$, then $(R_a, R_b) = R_{(a,b)} = 1$,
$R_{ab} = R_aX = R_bY$, $X = cR_b$, where $c \geq 1$ is integer. $R_{ab} = cR_aR_b$.

2) Let $(a, b) = d > 1$, $a = a_1d$, $b = b_1d$, $(a_1, b_1) = 1$, $a_1 \geq 1$, $b_1 \geq 1$. As $gcd(R_a, R_b) = R_{(a,b)}$, we receive equality

$$R_a = R_{(a,b)}X, R_b = R_{(a,b)}Y, \tag{10}$$

where $(X, Y) = 1$.

Further, $R_{ab} = R_aA = R_bB = XAR_{(a,b)} = YBR_{(a,b)}$, $XA = YB$, $A = Yz$, $B = Xz$, $z \geq 1$ is integer. Then $R_{ab} = XYR_{(a,b)}z$, $R_{ab} = zR_aR_b/R_{(a,b)}$. We have proved, that $R_aR_b/R_{(a,b)} \leq gcd(R_{ab}, R_aR_b)$.

Let's assume, that $gcd(R_{ab}, R_aR_b) = R_aR_b$, then $R_{ab} = zR_aR_b$, where $z \geq 1$ is integer. Let's consider equalities

$$R_{ab} = R_aA = R_bB,$$

where

$$A = 10^{a(b-1)} + 10^{a(b-2)} + \ldots + 10^a + 1,$$
$$B = 10^{b(a-1)} + 10^{b(a-1)} + \ldots + 10^b + 1.$$

Since $A = R_bz$, $B = R_az$, $10^a \equiv 1(mod\, R_{(a,b)})$, $10^b \equiv 1(mod\, R_{(a,b)})$, then $A \equiv B \equiv 0(mod\, R_{(a,b)})$, hence $a \equiv b \equiv 0(mod\, R_{(a,b)})$.

Thus, comparison $(a, b) \equiv 0(mod\, R_{(a,b)})$ or $d \equiv 0(mod\, R_d)$ is fair, that contradicts an obvious inequality

$$(10^x - 1)/9 > x, \tag{11}$$

where $x > 1$ is real. □

({★} **The Important corollary of the theorem 5**).
Number $R_{ab}/(R_aR_b)$ is integer when and only when $(a, b) = 1$, where $a \geq 1$, $b \geq 1$ are integers.

Let's quote some trivial statements for numbers repunit.

Lemma 1. *If $a = 3^nb$, $(b, 3) = 1$, then*

$$R_a \equiv 0(mod\, 3^n),\ but\, R_a \not\equiv 0(mod\, 3^{(n+1)}). \tag{12}$$

Proof. If $n = 1$, then $R_a = R_3 B$, where $B = 10^{3(b-1)} + \ldots + 10^3 + 1$, $R_3 \equiv 0(mod\,3)$, $B \equiv b \not\equiv 0(mod\,3)$. Thus,
$R_a \equiv 0(mod\,3)$, but $R_a \not\equiv 0(mod\,3^2)$.

Let comparisons (12) be proved for $n \leq k-1$. We shall consider $a = 3^k b$, $(b, 3) = 1$. Then $R_a = R_{3^{k-1}b} A$, where $A = 10^{3^{k-1}b2} + 10^{3^{k-1}b} + 1$.

$R_{3^{k-1}b} \equiv 0(mod\,3^{k-1})$, but $R_{3^{k-1}b} \not\equiv 0(mod\,3^k)$, $A \equiv 0(mod\,3)$, but $A \not\equiv 0(mod\,3^2)$. □

Lemma 2. *If $n \geq 0$ is integer, then*

$$r_n = 10^{11^n} + 1 \equiv 0(mod\,11^{n+1}),\ but\ r_n \not\equiv 0(mod\,11^{n+2}). \qquad (13)$$

Proof. $r_0 = 11 \equiv 0(mod\,11)$, but $r_0 = 11 \not\equiv 0(mod\,11^2)$.
$r_1 = 10^{11} + 1 \equiv 0(mod\,11^2)$, but $r_1 \not\equiv 0(mod\,11^3)$.

Let's make the inductive assumption, that formulas (13) are proved for $n \leq k-1$, where $k-1 \geq 1$, $k \geq 2$. Let $n = k$, then
$r_k = 10^{11^k} + 1 = (10^{11^{k-1}})^{11} + 1 = r_{k-1}A$, where

$$A = 10^{11^{k-1}10} - 10^{11^{k-1}9} + 10^{11^{k-1}8} - 10^{11^{k-1}7} + 10^{11^{k-1}6} -$$
$$- 10^{11^{k-1}5} + 10^{11^{k-1}4} - 10^{11^{k-1}3} + 10^{11^{k-1}2} - 10^{11^{k-1}} + 1. \qquad (14)$$

Since, due to the inductive assumption $10^{11^{k-1}} \equiv -1(mod\,11^k)$, where $k \geq 2$, then $A \equiv 11(mod\,11^k)$. Then $A \equiv 0(mod\,11)$, but
$A \not\equiv 0(mod\,11^2)$. Thus, we receive, that $r_k \equiv 0(mod\,11^{k+1})$, but
$r_k \not\equiv 0(mod\,11^{k+2})$. □

Lemma 3. *For an integer $a \geq 1$, the following statements are true :*

(1) If a is odd, then $R_a \not\equiv 0(mod\,11)$.

(2) If $a = 2(11^n)b$, $(b, 11) = 1$, then

$$R_a \equiv 0(mod\,11^{n+1}),\ but\ R_a \not\equiv 0(mod\,11^{n+2}). \qquad (15)$$

Proof. If a is odd, then $R_a \equiv 1(mod\,11)$. If $a = 2(11^n)b$, $(b, 11) = 1$, then $R_a = ((10^{2(11)^n})^b - 1)/9 = R_{11^n} \cdot r_n \cdot A$, where $r_n = 10^{11^n} + 1$,
$A = 10^{2(11^n)(b-1)} + \ldots + 10^{2(11^n)} + 1$. $R_{11^n} \not\equiv 0(mod\,11)$,
$A \equiv b \not\equiv 0(mod\,11)$. Then validity of the statement (2) of lemma 3 follows from lemma 2. □

({★} **The assumption: the general formula for $gcd(R_{ab}, R_a R_b)$).**
If $a \geq 1$, $b \geq 1$ are integers, $d = (a, b)$, where $d = 3^L \cdot 11^S \cdot c$, $(c, 3) = 1$, $(c, 11) = 1$, $L \geq 0$, $S \geq 0$, then equalities are true :

— if c is an odd number, then

$$gcd(R_{ab}, R_a R_b) = ((R_a R_b)/R_{(a,b)}) \cdot 3^L, \qquad (16)$$

— if c is an even number, then

$$gcd(R_{ab}, R_a R_b) = ((R_a R_b)/R_{(a,b)}) \cdot 3^L \cdot 11^S. \qquad (17)$$

Let's give another two obvious statements in which divisors of numbers repunit are studied, as degrees of prime number.

Lemma 4. *If p, q are prime numbers and $R_p \equiv 0 (mod\, q)$, but $R_p \not\equiv 0 (mod\, q^2)$, then statements are true :*

(1) For any integer r, $0 < r < q$, $R_{pr} \not\equiv 0 (mod\, q^2)$.

(2) For any integer n, $n \geq 1$, $R_{p^n} \not\equiv 0 (mod\, q^2)$.

Proof. 1) $R_{pr} = R_p \cdot \widehat{R}_{pr}$, where $\widehat{R}_{pr} = 10^{p(r-1)} + 10^{p(r-2)} + + \ldots + 10^p + 1$. If $R_{pr} \equiv 0 (mod\, q^2)$, then $\widehat{R}_{pr} \equiv 0 (mod\, q)$, $r \equiv 0 (mod\, q)$. Have received the contradiction.

2) If $n > 1$ found such that $R_{p^n} \equiv 0 (mod\, q^2)$, then from (7) follows $(R_{p^n}/R_p, R_p) = 1$. Have received the contradiction. □

Lemma 5. *If p, q are prime numbers and $R_p \equiv 0 (mod\, q)$, then*
$R_{pq^n} \equiv 0 (mod\, q^{n+1})$.

Proof. Since $R_{pq} = R_p \cdot \widehat{R}_{pq}$, where $\widehat{R}_{pq} = 10^{p(q-1)} + + 10^{p(q-2)} + \ldots + 10^p + 1$, then $\widehat{R}_{pq} \equiv 0 (mod\, q)$, $R_{pq} \equiv 0 (mod\, q^2)$. Let's assume that $R_{pq^{n-1}} \equiv 0 (mod\, q^n)$. Then
$R_{pq^n} = R_{pq^{n-1} \cdot q} = R_{pq^{n-1}} \cdot \widehat{R}_{pq^{n-1} \cdot q}$, where
$\widehat{R}_{pq^{n-1} \cdot q} = 10^{pq^{n-1} \cdot (q-1)} + 10^{pq^{n-1} \cdot (q-2)} + \ldots + 10^{pq^{n-1}} + 1 \equiv 0 (mod\, q)$,
$R_{pq^n} \equiv 0 (mod\, q^{n+1})$. □

4 Problem of simplicity of initial numbers

Let's consider the problem of simplicity of initial numbers $E_{n,k}$, where $k \geq 0$, $n \geq 0$.

If $k = 0$, then $E_{n,0} = R_{n+1}$. Thus, simplicity of numbers $E_{n,0}$ – is known problem of prime numbers repunit R_p, where p is prime number.

If $n = 1$, then $E_{1,k} = 1\{0\}_k 1 = 10^{k+1} + 1$. As number $E_{1,k}$ can be prime only when $k + 1 = 2^m$, $m \geq 0$ is integer, then we come to the known problem of simplicity of the generalized Fermat numbers $f_m(a) = a^{2^m} + 1$ for $a = 10$. Generalized Fermat numbers nave been define by Ribenboim [5] in 1996, as numbers of the form $f_n(a) = a^{2^n} + 1$, where $a > 2$ is even.
The generalized Fermat numbers $f_n(10) = 10^{2^n} + 1$ for $n \leq 14$ are prime only if $n = 0, 1$. $f_0(10) = 11$, $f_1(10) = 101$.
Theorem 6. *Let $n > 1$, $k > 0$. If any of conditions*

(1) n number is odd,

(2) k number is odd,

(3) $n + 1 \equiv 0 (mod\, 3)$,

(4) $(n + 1, k + 1) = 1$,

is true, then number $E_{n,k}$ is compound.

Proof. 1) $n + 1 = 2t$, $t > 1$. Then $E_{n,k} = E_{t-1,k} \cdot (10^{t(k+1)} + 1)$, where $t > 1$, $t - 1 \geq 1$. As $E_{t-1,k} > 1$, then $E_{n,k}$ is compound number.

2) Let k be an odd number. Due to the proved condition (1) we count that number $(n + 1)$ is odd. $k + 1 = 2t \geq 2$, $t \geq 1$. Further,

$$E_{n,k} = E_{n,t-1} \cdot ((10^{(n+1)t} + 1)/(10^t + 1)),$$

where $n > 1$, $t - 1 \geq 0$, $E_{n,t-1} > 1$, number $(10^{(n+1)t} + 1)/(10^t + 1) > 1$ is integer.

3) If $n+1 \equiv 0 (mod\, 3)$, then $E_{n,k} \equiv 0 (mod\, 3)$, $E_{n,k} > 11$.

4) Let $n > 1$, $k \geq 1$, $(n+1, k+1) = 1$, then

$$E_{n,k} = R_{(n+1)(k+1)}/R_{(k+1)} = R_{(n+1)} \cdot (R_{(n+1)(k+1)}/(R_{k+1} \cdot R_{n+1})).$$

Due to the theorem 5 number $z = R_{(n+1)(k+1)}/(R_{k+1} \cdot R_{n+1})$ is integer. Further,
$z > (10^{(n+1)(k+1)} - 1)/(10^{n+k+2}) = 10^{nk-1} - 1/(10^{n+k+2})$, $nk - 1 \geq 1$, thus, $z > 1$. $\square$

Question of simplicity of initial numbers under conditions, when $(n+1, k+1) > 1$, $(n+1)$ number is odd, $(k+1)$ number is odd, $n+1 \not\equiv 0 (mod\, 3)$, remains open.

In particular, it is interesting to considerate numbers $E_{p-1,p-1} = R_{p^2}/R_p$, where p is prime number. For $p < 100$ numbers $E_{p-1,p-1}$ are compound.

5 The open problems of numbers repunit

The known problem of numbers repunit remains open.

Problem 1 ((Prime repunit numbers[3])). *Whether there exists infinite number of prime numbers R_p, p–prime number ?*

Problem 2. *Whether all numbers R_p, p–prime number, are numbers free from squares ?*

The author has checked up for $p < 97$, that numbers R_p are free from squares. Another following open questions are interesting :

Problem 3. *If number R_p is free from squares, where $p > 3$ is prime number, whether will number n, be found such what number R_{p^n} contains a square ?*

Problem 4. *p is prime number, whether there are simple numbers of a kind $E_{p-1,p-1} = R_{p^2}/R_p$?*

The author has checked up to $p \leq 127$, that numbers $E_{p-1,p-1}$ is compound. It is known, that R_p divide by number $(2p+1)$ for prime numbers $p = 41, 53$, R_p divide by number $(4p+1)$ for prime numbers $p = 13, 43, 79$. There appears a question:

Problem 5. *Whether there is infinite number of prime numbers p, such that R_p divide by number $(2p+1)$ or is number $(4p+1)$?*

(**The remark**). *If the number $p > 5$ Sophie Germain prime (i.e. number $2p+1$ is prime too), then either R_p or $R_p^+ = (10^p + 1)/11$ divide by number $(2p+1)$.*

6 The conclusion

Leonhard Euler, professor of the Russian Academy of sciences since 1731, **has paid mathematics forever !** Euler's invisible hand directs the development of concrete mathematics for more than 200 years.

Euler's titanic work which has opened a way to freedom to mathematical community, admires. The pleasure caused by Euler's works warms hearts.

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